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Review
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VOLATILITY AND KURTOSIS OF DAILY STOCK RETURNS AT MSE

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Abstract
Prominent financial stock pricing models are built on assumption that asset returns follow a normal (Gaussian) distribution. However, many authors argue that in the practice stock returns are often characterized by skewness and kurtosis, so we test the existence of the Gaussian distribution of stock returns and calculate the kurtosis of several stocks at the Macedonian Stock Exchange (MSE). Obtaining information about the shape of distribution is an important step for models of pricing risky assets. The daily stock returns at Macedonian Stock Exchange (MSE) are characterized by high volatility and non-Gaussian behaviors as well as they are extremely leptokurtic. The analysis of MSE time series stock returns determine volatility clustering and high kurtosis. The fact that daily stock returns at MSE are not normally distributed put into doubt results that rely heavily on this assumption and have significant implications for portfolio management. We consider this stock market as good representatives of emerging markets. Therefore, we argue that our results are valid for other similar emerging stock markets.

Keywords: models, leptokurtic, investment, stocks.

Jel Classification: G1; G12

INTRODUCTION

For a long time Gaussian models (Brownian motion) were applied in economics and finances and especially to model of stock prices return. However, in the practice, real data for stock prices returns are often characterized by skewness, kurtosis and have heavy tails. Many financial economists argue and provide empirical evidences that some stock prices returns are not distributed by Gaussian distribution because of fat tails and strong asymmetry and argue that they are leptokurtic.

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We can find in finance literature many different models that have been proposed to deal with these departures from Gaussian distribution like following: stable symmetric (stable Paretian) distributions for stock returns, the Student t, the generalized beta distribution of the second kind and mixtures of Gaussian distribution (Mills 1995). All above mentioned three competing hypothesis have subsequently been used to explain this observed tail behavior of security returns.

Kurtosis as statistical measure has significant importance for investors, because represent the possibility of the price of stocks to change significantly (up or down from current levels). Obtaining information about the shape of distribution is an important step for models of pricing risky assets where distribution and estimates of volatility are used as inputs (Ivanovski, Narasanov, and Ivanovska 2015).

In this paper we test the existence of the Gaussian distribution of stock returns and calculate the kurtosis of several stocks at the Macedonian Stock Exchange (MSE). MSE is emerging stock market with very low liquidity. The aim of this study is to determine the kurtosis of several stocks at MSE and to test if this measure provides signals for future behavior of stock prices at MSE. The basic idea is to test functionality of this measure as additional parameter for the stock pricing process. The purpose of this paper is also to contribute to the debate concerning the relationship between returns and volatility for the emerging markets of the Central and Eastern Europe.

We address the following research questions: Are there notable differences between kurtosis of stocks as the parameters of stock pricing models for MSE? What is the level of volatility on MSE? What is the practical use of kurtosis as indicator for stock prices movements at MSE?

While we draw our conclusions from the historical data on MSE, we consider this stock market as good representatives of emerging markets. Therefore, we argue that our results are valid for other similar emerging stock markets.

The remainder of the paper is organized as follows. In Section I we give summary of literature overview concerning stock return distributions, presenting different models from Gaussian distribution, like the stable symmetric (stable Paretian) distributions for stock returns, the Student t, the generalized beta distribution of the second kind and mixtures of Gaussian distribution as well as explanation of kurtosis as a statistical measure. In second part of this section we address literature overview about volatility for MSE. Section II describes tools used in research for derivation of stochastic parameters. In Section III we present the results on the derivation of stochastic parameters from the analysis of historical data from MSE. Section IV gives conclusions and possible directions for future research.

1. LITERATURE REVIEW

At the begging of the 20th century Louis Bachelier (1900) published the first paper where advanced mathematics was used in the study of finance. His model for stochastic process, now called Brownian motion, has since than become the dominant model for stock pricing processes of modern finance. Stock valuation models are built on assumption for the normal (Gaussian) distribution: Markowitz Portfolio Theory (Markowitz 1952; 1959), Capital Asset Pricing Model (Sharpe 1964), Option Pricing Theory (Black and Scholes 1973). That overcome previously widely accepted hypothesis that rational investors' preferences can be analyzed only in terms of expected returns and risk measured by the variance and standard deviation of the return distribution.

However, risk models are widely used in stock valuation in order to provide a measure of risk necessary for portfolio selection and optimization, risk management, and derivatives pricing.

A risk model is typically a combination of a probability distribution model and a risk measure and provide following: first, calculate returns' temporal dynamics, such as autocorrelations, volatility clustering, and long memory, second, employs a distributional assumption flexible enough to accommodate various degrees of skewness and heavy-tails, third, is scalable and practical and can be extended to a multivariate model covering a large number of assets (Rachev and Mittnik 2000).

Markowitz Portfolio Theory (MPT) used an asset's return as a normally distributed function. This theory calculate risk as the standard deviation of return as well the return of a portfolio as the weighted combination of the assets' returns. MPT suggest possibility to reduce the total variance of the portfolio return by adding different assets with returns that are not perfectly positively correlated. This theory also has basic assumption that investors are rational and markets are efficient (Chamberlain 1983). Although MPT was developed in the 1950s, through the 1970s it gained popularity and was considered as a basis for portfolio management. However, since then, many theoretical papers and practical considerations provided many evidences against it esspecially that financial returns do not have a Gaussian distribution or any symmetric distribution. In many empirical studies it was noticed that returns of stocks (indexes, funds) are badly fitted by Gaussian distribution because of heavy tails and strong asymmetry (Mandelbrot 1960; 1963). Some authors (Mandelbrot and Hudson 2006) elaborates that random walk and Gaussian daily returns simply do not correspond to reality, and grossly underestimates the risk of huge market swings.

Fama (Fama 1965) reported that daily returns of stocks on the Dow Jones Industrial Average (DJIA) display more kurtosis than permitted under the normality hypothesis. Since that early work of Fama, it has typically been found that daily returns display more kurtosis than that permitted under the assumptions of normality, while skewness has also been prevalent (Mills 1995). The expected returns and variances are almost always estimated using past returns rather than future returns.

The bias towards positive or negative returns is represented by the skewness of the distribution. If distribution is positively skewed, there is higher probability of large positive returns than negative returns. The shape of the tails of the distribution is measured by the kurtosis of the distribution; fatter tails lead to higher kurtosis. (Damodaran 2006).

Normal distributions are symmetric (no skewness) and defined to have a kurtosis of zero. When return distributions take this form, the characteristics of any investment can be measured with two variables - the expected return, which represents the opportunity in the investment, and the standard deviation or variance, which represents the level of danger. (Damodaran 2006)

Investors who are usually risk averse prefer positive skewed distributions to negatively skewed ones as well as distributions of returns with a lower possibility for significant price changes (lower kurtosis) to those with a higher possibility of jumps (higher kurtosis). The coefficient of skewness gives information on the distribution of the returns of each stock (Skrinjaric 2014). When the distribution is positively skewed, it means that returns are greater than the expected. Most of the models have assumption that investors prefer stocks with the positive skewness return distribution.

Kurtosis measures the degree of a distribution expressed as fat tails. Most of the investors are risk-averse which means that they prefer a distribution with low kurtosis, or we can explain differently as returns that are not far away from the mean. For normal distribution an excess kurtosis has to be equal to 0. When we have a case of a positive skewness, it means to become possible to have a high excess kurtosis and not to have extreme negative returns in the future as well as that the extreme returns will only be positive. This can happen when the skewness is positive. When we have negative skewness, investors can face the extreme negative returns due to the impact of a high excess kurtosis. In a case of return distribution with skewness lower than -1 and an excess kurtosis higher than 1, there is high probability to face sudden high negative returns increases (Ivanovski, Narasanov, and Ivanovska 2015).

Finance literature proposed many different models to deal with these departures from Gaussian distribution, like the stable symmetric (stable Paretian) distributions for stock returns, the Student t, the generalized beta distribution of the second kind and mixtures of Gaussian distribution (Mills 1995). They explain this observed tail behavior of security returns.

In probability theory, a random variable is said to be stable (or to have a stable distribution) if it has the property that a linear combination of two independent copies of the variable has the same distribution, up to location and scale parameters (Mills 1995). Most of the researches in the finance literature are focused on stable distributions. However, we are witnesses that practical implementation of stable distributions to risk modeling has recently been developed and implemented in practice as a result of high complexity for appropriate fitting and simulating stable models. In order to make distinction between Gaussian and non-Gaussian stable distributions (so called stable Paretian, Lévy stable or α-stable distributions) we have to emphasize that stable Paretian tails decay more slowly than the tails of the normal distribution and can be used for prediction as well as to describe the extreme events present in the data (Rachev and Mittnik 2000). Student's t distribution as well as stable Paretian distributions have a parameter responsible for the tail behavior, called tail index or index of stability (Barndorff-Nielsen and Shephard 2001).

Stable Paretian distributions (Mittnik et al. 1999) have attractive properties for empirical modeling in finance, because they include the normal distribution as a special case but also allow for heavier tails and skewness. Authors argue that the stable Paretian distribution gives rise to more realistic distributional models for returns on financial assets, such as stocks, futures or foreign exchange, because financial return data are typical heavy tailed and often skewed. The stable Paretian family allows for such phenomena without ruling out the normal distribution (Mittnik et al. 1999).

A *t*-test is any statistical hypothesis test in which the test statistic follows a Student's *t* distribution. It can be used to determine if two sets of data are significantly different from each other, and is most commonly applied when the test statistic would follow a normal distribution if the value of a scaling term in the test statistic were known (Andersen et al. 2000).

Similar to the Student's t distribution, stable distributions can be represented as mixtures of other distributions. The price and return dynamics can be considered under two different time scales- the physical time and an intrinsic (also called market) time. The intrinsic time is best thought of as the cumulative trading volume process which measures the cumulative trading volume of the transactions and can be considered as a measure of market activity (Rachev and Mitnik 2000).

Financial market volatility is central to the theory and practice of asset pricing, asset allocation, and risk management. Volatility, however, is only one of the distributional moments that can provide a stylized representation of returns (Gabrielsen et al. 2012).

One of the popular models used for volatility measurement is the equally weighted moving average model. This framework assumes that the *N*-period historic estimate of variance is based on an equally weighted moving average of the *N*-past one-period squared returns (Gabrielsen et al. 2012). However, under this formulation all past squared returns that enter the moving average are equally weighted and this may lead to unrealistic estimates of volatility (Gabrielsen et al. 2012).

In order to overcome mentioned inconsistency the exponentially weighted moving average (EWMA) framework was proposed by J.P Morgan's Risk Metrics TM that assigns geometrically declining weights on past observations with the highest weight been attributed to the latest (i.e. more resent) observation (Gabrielsen et al. 2012). By assigning the highest weight to the latest observations and the least to the oldest the model is able to capture the dynamic features of volatility (Barndorff-Nielsen and Shephard 2001).

Other approaches in this direction are the very often used ARCH and GARCH model proposed by Engle and Bollerslev (1986). Bollerslev is author of the Autoregressive Conditional Heteroscedasticity (ARCH), which models the variance of a time series by conditioning it on the square of lagged disturbances and the latter generalizes the ARCH model by considering the lagged variance as an explanatory variable. Some authors (Gabrielsen et al. 2012) provides an insight to the time-varying dynamics of the shape of the distribution of financial return series by proposing an exponential weighted moving average model that jointly estimates volatility, skewness and kurtosis over time using a modified form of the Gram-Charlier density in which skewness and kurtosis appear directly in the functional form of this density (Gabrielsen et al. 2012).

Concerning volatility at MSE, in his paper (Kovacic 2007) analyzes a volatility of stock market index in Macedonia. In fact this study for the first time present results for volatility at MSE. MSE was not previously considered in the volatility literature. He uses formal statistical tests and graphs of the MBI-10 returns, corresponding functions and estimated GARCH-type models and derive several conclusions: a) that large changes in returns at MSE tend to be followed by large changes and small changes tend to be followed by small changes, which means that volatility clustering is observed in the Macedonian financial returns series; b) the results related to the relationship between returns and conditional volatility can be regarded as quite robust across the models and alternative error distributions; c) the forecasting performance of asymmetric GARCH models (GJR and TGARCH in particular) is better than symmetric GARCH models, but with little gain (Kovacic 2007). However, Kovacic also emphasizes some certain reservations about his conclusion, due to the fact that the time series of returns is quite short.

2. TOOLS FOR DERIVATION OF STOCHASTIC PARAMETERS

The aim of this study is to investigate the nature and dynamics of the shape of the distribution of the stock daily returns over time at MSE in order to determine if they have Gaussian distribution. We use an Exponentially Weighted Moving Average and Rolling Window Moving Average Estimator to determine the level of volatility of daily stock returns, and then calculate kurtosis to test the accuracy of the assumption that the stock returns are normally distributed.

Valuation of financial instruments depends strongly on volatility estimates. There are two broad approaches: historical and implied volatility. The historical approach assumes that past holds predictive power for the future. On the other hand, implied volatility is calculated from the assumption that the market prices implicitly contain a consensus estimate of volatility (Andersen et al. 2000).

Even for historical volatility there exist several models. Different models can result in different estimates. EWMA and GARCH models analyses the dynamic structure of volatility, and provide accurate forecasting for future behavior of risk. Therefore, they should provide more accurate results than constant, rolling window volatility models.

Historical approaches have two steps in common: (i) Calculate the series of periodic returns; (ii) Apply a weighting scheme.

First, for each day, we take the natural log of the ratio of stock prices.

$$\mu_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \tag{1}$$

This produces a series of *m*-1 daily returns e.g. from μ_2 to μ_m, if there are price measurements for *m* days S_1, S_2, S_3, …, S_m. Daily returns are expressed in continually compounded terms. Then we calculate the average return for the whole measurement period.

$$\mu = \frac{1}{m-1}\sum_{i=2}^{i=m}\mu_i \tag{2}$$

Second step estimates the variance from the same series of daily returns, as shown next.

2.1. Rolling Window Moving Average Estimator

The historical or *n*-period rolling window moving average estimator of the volatility corresponds to the standard deviation and it is given by the following expression

$$\hat{\sigma}_{t+1} = \sqrt{\frac{1}{n}\sum_{s=t-n+1}^{t}(\mu_s - \mu)^2} \tag{3}$$

where r_s is the return of the asset at period *s* and *m* is the mean return of the asset.

The size *n* is critical when one considers the effect of an extremely high or low observation in the sense that the smaller the size of the window, the bigger the effect on volatility.

The weakness of this approach is that all returns have the same weight and yesterday's (very recent) return has no more influence on the variance than last month's return.

2.2. Exponential Wighted Moving Averages Estimator

The problem of Rolling Window Moving Average Estimator is fixed by using the exponentially weighted moving average (EWMA), in which more recent returns have greater weight on the variance. EWMA calculations is given by the square root of

$$\hat{\sigma}_{t+1}^2 = \lambda \hat{\sigma}_t^2 + (1-\lambda)(\mu_t - \mu)^2 \tag{4}$$

where λ is the decay factor (smoothing constant). This method uses the weights that are geometrically declining, so the most recent observation has more weight compared to older ones. This weighting scheme helps to capture the dynamic properties of the data. Commonly, the smoothing constants are 0.94 for daily data and 0.97 for monthly data (Suganuma 2000).

2.3. Kurtosis

Kurtosis (Kenney and Keeping 1951) characterizes the relative peakedness or flatness of a distribution compared with the normal distribution. For a random variable x kurtosis is defined as

$$\mathrm{Kurt}[x] = \frac{E\left[(x-\bar{x})^4\right]}{\sigma^4} - 3 \tag{5}$$

where $E\left[(x-\bar{x})^4\right]$ is the fourth moment around the mean, and σ is the standard deviation of x. For a data series e.g. daily returns $\{\mu_i\}$, kurtosis $\mathrm{Kurt}[\mu_i]$ is calculated as

$$\mathrm{Kurt}[\mu_i] = \left(\frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum_{i=2}^{i=m} \frac{(\mu_i - \mu)^4}{\sigma^4} \right) - \frac{3(n-1)^2}{(n-2)(n-3)} \tag{6}$$

Distributions with zero kurtosis are called mesokurtic. Normal distribution has zero kurtosis. Distributions with high kurtosis distribution are called leptokurtic, and tend to have a distinct peak near the mean, decline rather rapidly, and have heavy tails. Distributions with negative kurtosis (platykurtic) have a flat top near the mean and shorter, thinner tails.

In the following sections histograms will be calculated for daily stock returns, and then kurtosis will be used to measure the assumption that the stock returns are normally distributed.

3. ANALYSIS OF MARKET DATA

In this paper we test the accuracy of assumptions of the Gaussian distribution of stock returns and calculate the kurtosis of stocks on Macedonian stock exchange. Statistical analysis of historical data on stock prices from Macedonian stock markets is given.

MSE is an emerging stock market with very low liquidity. The aim is to determine the model parameters for various models for the stock pricing process.

While we draw our conclusions from the historical data on MSE, we consider this stock market as good representative of emerging markets. Therefore, we argue that our results are valid for other similar stock markets.

Following table gives stochastic parameters for 10 stocks from MSE and its index MBI-10. Last row give the average value for the same parameters.

Table 1. Stochastic parameters for MSE

ISIN Code	Kurt	σ	σ_σ MA	σ_σ EWMA
ALK	4.10	0.0232	0.00856	0.00791
BESK	7.40	0.0270	0.01265	0.00903
GRNT	3.31	0.0271	0.01070	0.00825
KMB	4.54	0.0227	0.00875	0.00770
MPT	7.21	0.0256	0.00777	0.00806
REPL	27.27	0.0213	0.00937	0.00785
SBT	9.79	0.0210	0.00615	0.00684
STIL	32.55	0.0342	0.01907	0.01361
MTUR	8.72	0.0190	0.00551	0.00521
TPFL	5.41	0.0242	0.00874	0.00806
MBI	5.61	0.0167	0.00548	0.00636
MSE	10.54	0.0238	0.00934	0.00808

Note: Column headings are as follows: (1) Stock code; (2): Kurtosis of the daily return series; (3): Volatility of the daily return series; (4): Volatility of the volatility sequence calculated using rolling window moving average; and (5): Volatility of the volatility sequence calculated using exponentially weighted moving average.

Kurtosis values of individual stocks at MSE are presented on following histograms:

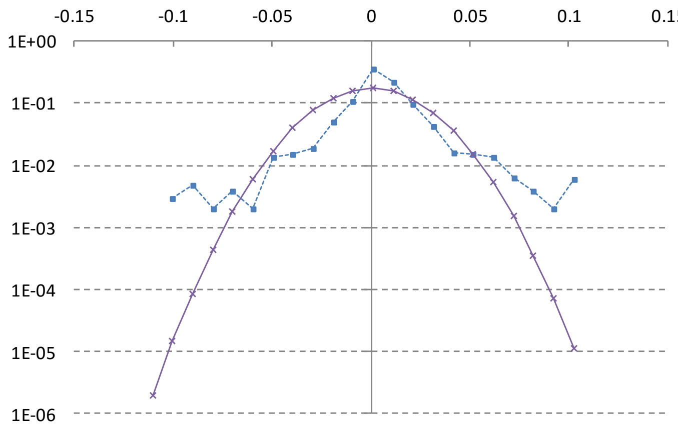

Figure 1. Histogram for ALK. Kurt = 4.10.

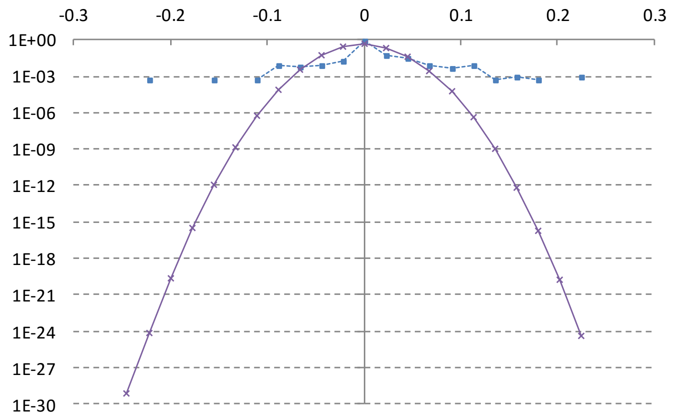

Figure 2. Histogram for REPL. Kurt = 27.27.

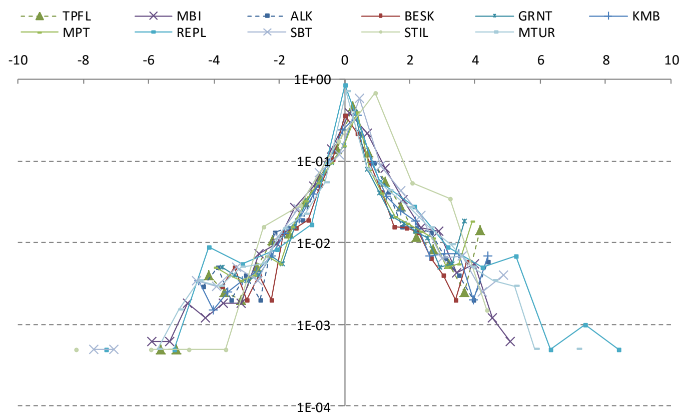

Figure 3. All histograms for daily stock returns at MSE.

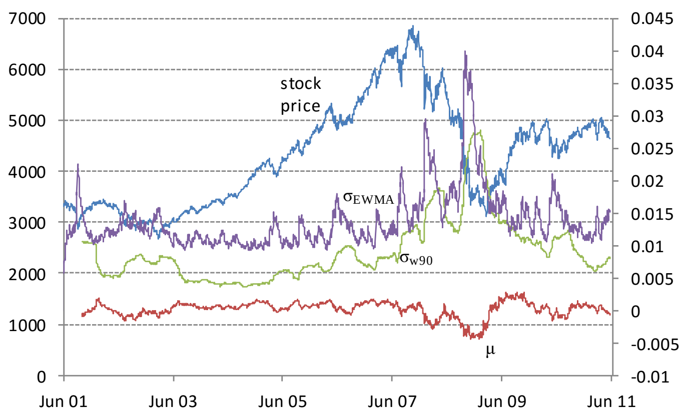

Figure 4. Histogram for stock price, daily return μ, 90-days rolling window moving average estimator of the volatility σ_{w90}, and EWMA estimator of the volatility σ_{EWMA} for MSE.

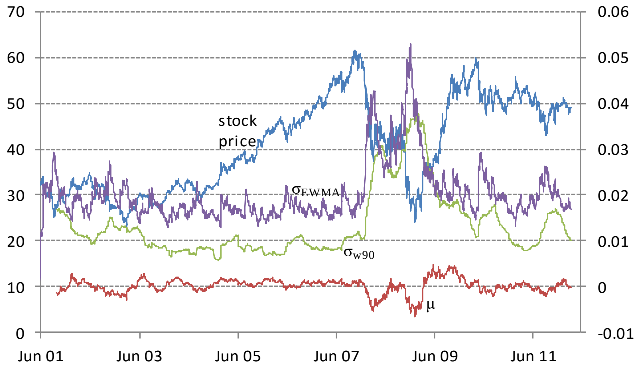

Figure 5. Histogram for stock price, daily return μ, 90-days rolling window moving average estimator of the volatility σ_{w90}, and EWMA estimator of the volatility σ_{EWMA}. for SBT.

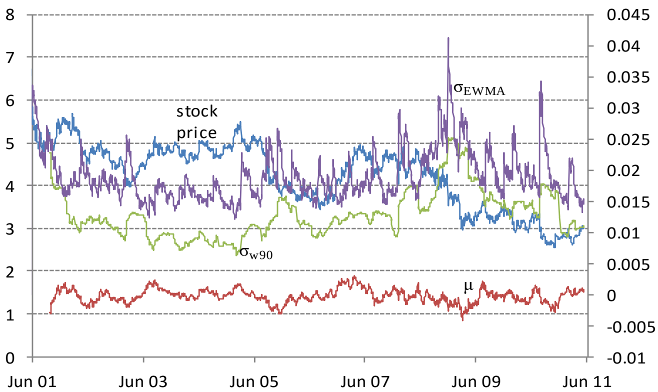

Figure 6. Histogram for stock price, daily return μ, 90-days rolling window moving average estimator of the volatility σ_{w90}, and EWMA estimator of the volatility σ_{EWMA} for GRNT.

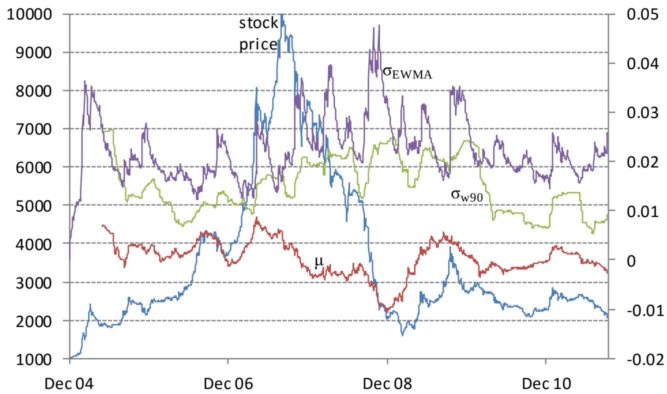

Figure 7. Histogram for stock price, daily return μ, 90-days rolling window moving average estimator of the volatility σ_{w90}, and EWMA estimator of the volatility σ_{EWMA} for MBI-10 index

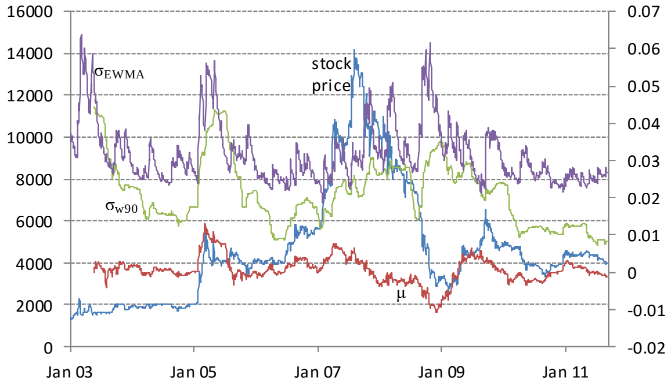

Figure 8. Histogram for stock price, daily return μ, 90-days rolling window moving average estimator of the volatility σ_{w90}, and EWMA estimator of the volatility σ_{EWMA} for ALK

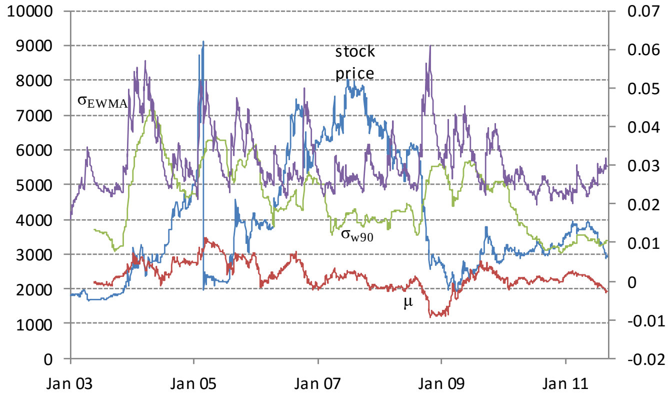

Figure 9. Histogram for stock price, daily return μ, 90-days rolling window moving average estimator of the volatility σ_{w90}, and EWMA estimator of the volatility σ_{EWMA} for KMB

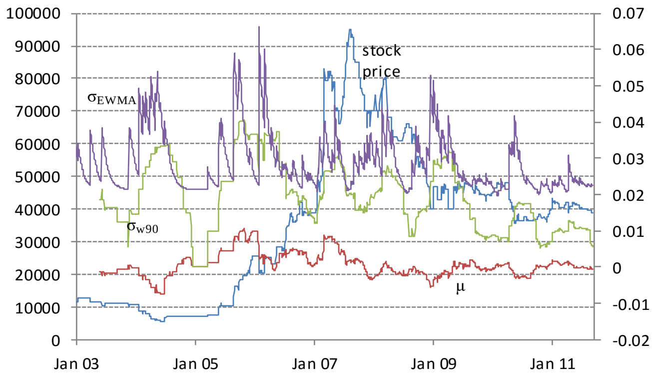

Figure 10. Histogram for stock price, daily return μ, 90-days rolling window moving average estimator of the volatility σ_{w90}, and EWMA estimator of the volatility σ_{EWMA} for REPL

Several conclusions can be made from Table 1 and histograms. It is obvious that histograms of the daily return series for all stocks from MSE are leptokurtic, with no exception. This means that significant variations in the daily prices are much more common than estimated by the normal distribution.

All MSE stocks have large kurtosis values. This means that large daily changes i.e. heavy tails occur more frequently on some MSE stocks. However, such changes are not immediately followed by the all other stocks on MSE, which results in reduced overall impact on MBI-10 changes. This explains why MBI-10 index has smaller kurtosis than individual stocks. The highest kurtosis have stocks at MSE with very small liquidity (REPL, STIL).

The impact of higher kurtosis for MSE stocks did not signalize significant stock price changes because in period 2011-2013 due to the bearish trend high drops or peaks of stock prices not happened. Comparing columns 4 and 5, one can readily see that the values for volatility of the volatility sequence are very similar when calculated using rolling window moving average and exponentially weighted moving average methods.

CONCLUSION

We can conclude that histograms of the daily return series for all stocks from MSE are leptokurtic, with no exception. This means that significant variations in the daily prices are much more common than estimated by the normal distribution.

Kurtosis have large values for MSE stocks. This means that large daily changes i.e. heavy tails occur more frequently on some MSE individual stocks. However, such changes are not immediately followed by the all other stocks on MSE, which results in reduced overall impact on MBI-10 changes. This explains why MBI-10 index has smaller kurtosis than individual stocks. Highest kurtosis have stocks at MSE with the smallest liquidity (REPL, STIL).

The impact of higher kurtosis for MSE stocks did not signalize significant stock price changes because in period 2011-2013 due to the bearish trend high drops or peaks of stock prices not happened. We find that the values for volatility of the volatility sequence are very similar when calculated using rolling window moving average (RWMA) and exponentially weighted moving average methods (EWMA). We can see from the histogram of daily returns that they do not follow the Gaussian curve and that they are with heavy tails. The distribution of the MBI-10 returns is characterized not only by heavy tails, but also by a high peakedness at the center. This findings confirm the already mentioned conclusion of Kovacic paper (Kovacic 2007).

The empirical results show the following: (i) the Macedonian stock returns time series display stylized facts such as volatility clustering and high kurtosis (Kovacic 2007); (ii) the histograms of returns and a Gaussian density shows that numerous returns are highly unlikely to have the Gaussian distribution and are with heavy tails.

This study outlines directions for future researches that could be investigated to improve the modeling and volatility forecasts of the Macedonian stock market returns. Due to the fact that we use ten-year time series of returns (2005–2014), longer time series would allow estimation with greater precision.

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