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THE EPISTEMOLOGICAL ROLE OF S&P 500 SIGNAL'S NON-STATIONARITY ON INVESTORS' DYNAMIC SENTIMENT FORMATION: EVIDENCE FOR INVESTORS' PROSPECT THEORY PREFERENCES

Gojart Kamberi [1]

Abstract
Financial signals' abrupt spectral changes in temporal scale correspond to the perception bias on risk which is a well-known epistemologically decision-making factor in behavioral finance research. In this paper, we explored the epistemological role of S&P 500 signal's non-stationarity on investors' sentiment formation, by analyzing the time-frequency dependency of investors' sentiment on the S&P 500 through a cross wavelet transform analysis to further explore their wavelet coherence (co-movements). Results indicate that S&P 500 signal's abrupt spectral changes across time do have a statistically significant impact on the investors' sentiment formation. Moreover, a comparative analysis of the wavelet coherences between the Bullish/Bearish market sentiments and the S&P 500's signal, reveals the investors' dynamic (historical) epistemological bias toward risk, which corresponds with the dynamic prospect theory investment preferences.

Keywords**:** signal, sentiment, time, frequency, epistemology

JEL classification: G41, D87

INTRODUCTION

Investors are in a continuous perceptual processing and search effort for financial information. Regardless of the available computational resources to tackle the problem of extrapolation, both in time and frequency domain, there exists also a natural tendency of addressing extrapolation through the plain human perception resources. In the time domain, investors perceptually extrapolate the signal by assuming that the mean and variance signal do not change across time (Marshall 1994), whereas on the frequency domain humans perceptually extract and learn the underlying frequencies that constitute the signal (Bezuidenhout and van Vuuren 2021). This generalization of the concept "signal" corresponds to inclusivity of all human sensory perception where each sensory information received regardless of whether it is gustation, tactile, auditory, olfaction or visual signal, can be generalized throughout a time-frequency domain. Furthermore, the human's assumption of stationarity serves as a reductionism mechanism to focus on the essence of the signal which do correspond with the contextuality of the computationally financial signal decomposition into trend, seasonality and noise.

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A financial signal whose non-stationarity property does not allow investors to make use of its continuously changing statistical properties (mean, variance, and covariance) as to build a stable representation of its change over time, requires a different processing strategy compared to the extrapolation methods whose basic assumption is the signal's stationarity. Referring to different past time points of the financial signal when extrapolating, serves the investors to build both, the sense of measurement and the sense of continuity in the act of measurement. The latter two represent epistemologically two different properties of the financial signal extrapolation, where the measurement itself assumes stationarity, and continuity in the act of measurement assumes non-stationarity. How often and how long a particular above (or below) average price level of the S&P 500's signal is about to be encountered, plays key importance on investors' investment preferences. If there is a historically constant longer-term above (or below) average trend, this means that the epistemological importance for such low frequency changes is low because the duration of the market being exposed to such trend more often and thus the market adapts to low frequency. On the other hand, if there is a historical instability of several abrupt above (or below) trend changes, the epistemological importance of such high frequency changes is high, even though the duration is long, thus the market cannot adapt to the abrupt changes.

Understanding non-stationary time series data behavior as different frequencies at different time steps, serves as a tool to discover dynamic spectral patterns along the financial signal. That is, investors must identify simultaneously the most dominant local frequency (for a selected period) of the financial signal, and then extrapolate its future spectrum evolution, which in a way means extrapolating the financial signal's non-stationarity across time. It is the additional epistemological value of the latter investors' behavioral intention what we are aiming to explore in this paper. Therefore, operationally, we define the cognitive dimensionality of non-stationary financial information processing into two basic cognitive processing scales: the time domain processing and the frequency domain processing. However, the continuality of non-stationary financial information across time implies that financial signal's frequency also is subject to change over time. That is, the need for an additional combined time-frequency domain exists, which is subject to the Heisenberg's uncertainty principle (Wilczok 2000). In the context of our study, Heisenberg's uncertainty principle explains the tradeoff that investors face with when they decide to allocate their attention during an extrapolation task, on the time domain, by trading off the focus from the frequency domain, and vice versa.

Previous behavioral finance research contemplates that investors' sentiment formation is an epistemologically biased attitude toward potential risk (Kahneman and Tversky 2013). The processing of financial information through perceptual mechanisms is supposed to generate reliable investors' probabilistic beliefs or expectations when extrapolating the future outcomes, for both optimistic and pessimistic investors' sentiments. However, perceptual mechanisms do react biased by favoring more the investors' probabilistic beliefs toward the pessimism (potential losses) than the probabilistic beliefs toward optimism (potential gains) (Ding, Charoenwong, and Seetoh 2004). Moreover, recent research on the dynamic version of prospect theory exists in neuroeconomic research (Fox and Poldrack 2009), which contemplates that the dynamics of uncertainty expressed through continuous dynamic probability distortions affects the decision maker's probability weighting functions. The additional core neuroeconomic theory which posits that decision makers update their beliefs through reward prediction error as a learning signal is the reinforcement learning theory (Shen et al. 2014). In our case, exploring for a potential change in a combined time-frequency dependency of investors' sentiments of Bullish and Bearish on S&P 500 signal, would offer evidence that such combined time-frequency dependency does indeed represent the investors' epistemologically biased (asymmetric) attitude towards future risk by which investors become less or more responsive on the epistemic value of S&P 500 signal's non-stationarity. As such we formulate our alternative hypotheses that:

H1. Investors' sentiment formation is sensitive on the combined time-frequency domain (non-stationarity) of the financial signal being extrapolated.

H2. Investors' sentiment of optimism has the tendency to become less responsive on the combined time-frequency domain (non-stationarity) of the financial signal compared to investors' sentiment of pessimism.

H3. Investors with prospect theory preferences do not weight dynamically perceived gains and losses the same.

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1. DATA AND METHODS

We use secondary data provided online from the American Association of Individual Investors (AAII) investor sentiment historical survey data (https://www.aaii.com/sentimentsurvey) on behalf of investor sentiment index which is historically measured through the AAII sentiment survey of the opinions of individual investors regarding the future price developments. The type of the data used are time series of investor sentiments in terms of Bullish, Neutral and Bearish and time series of S&P500 in the period 1987-2020 providing a time resolution of weekly basis.

In signal processing literature which is also highly adopted in financial signal processing literature (Akansu, Kulkarni, and Malioutov 2016), two main levels of signal analysis exist: the time domain analysis and the frequency domain analysis. These two levels of analysis, offer two different perspectives of addressing the financial signal's noise. Whereas in the time domain analysis, the signal's noise is one of the three components isolated after performing a signal decomposition procedure, in frequency domain analysis, the signal's constituent sine and cosine functions by setting an amplitude threshold are extracted and considered as the key cycles dominating the signal's behavior, while the sinusoid functions responsible for the noise are deducted as being below the frequency analysis threshold. The latter method is known as Fourier's transform (Duhamel and Vetterli 1990). In comparison with Fourier transform where the frequency domain is obtained while the time domain of the data is lost, the wavelet transform allows for a balanced information tradeoff between the time and frequency domains. Because this paper represents only an applied version of the CWT, for a more detailed mathematical treatment of the CWT than ours, we recommend to the reader additional reading sources (Pathak 2009).

The combined time-frequency information makes it possible to get insights for possible transient relations and structural changes when addressing a signal. The different periodic components of non-stationary signals change over time, and it is this change that accounts for power spectrum of the signal over time if we were to consider the S&P 500 signal as a data generating process. Furthermore, the oscillatory behavior of S&P 500 accounts historically for both short and long-term oscillations reflecting the markets' instabilities. Therefore, to map the S&P 500 signal into the time and frequency domain, in this paper, we decided to use the continuous wavelet transform (CWT), which is a tool widely used in financial econometrics (Eliasson 2018). The general form of CWT of a variable x(t) with respect to a wavelet function ψ is:

$$W_x(\tau, s) = \int_{-\infty}^{\infty} x(t)\left[\frac{1}{\sqrt{s}}\bar{\psi}\left(\frac{t-\tau}{s}\right)\right]dt,$$

where the two parameters τ and s are representing the wavelet function in terms of time-frequency domain, that is in terms of the location of the wavelet in temporal scale and the dilation factor measuring the short and long-run cycles. Whereas to test for wavelet coherence between the S&P 500 signal and investors' sentiment, we have used the wavelet coherence method which measures the correlation between two non-stationary financial signals (Ye et al. 2020).

2. RESULTS AND DISCUSSIONS

From the scalograms in Figure 1, we can observe that the frequency domain of investor sentiments' is widely spread and noisier across the time domain compared to the S&P 500 signal. This means that investor sentiments are less stable signals over time than the S&P 500 signal, that is, they are more uncertain on the combined time-frequency domain compared to the S&P 500 signal.

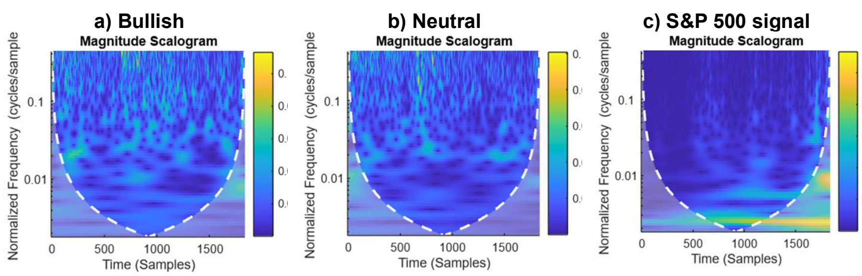

Figure 1. Wavelet transform of investors' sentiments' and S&P 500's signals:

Whereas the wavelet coherence between sentiment of Bullish and S&P 500 signal, and between sentiment of Bearish and S&P 500 signal is observed to be higher than between sentiment of Neutral and S&P 500 (Figure 2). This is in consistency with our first hypothesis. If we were to divide the whole-time scale (1837 weeks) into two equally distant temporal periods as past and recent periods (each containing approximately 900 weeks), we can further compare the frequency coherence in between Bullish & S&P 500 and Bearish & S&P 500 in the latter two periods. As such, in the past period, investors' Bullish sentiment has an observably greater co-movement with S&P 500 signal compared to their co-movement in the recent period. This can be interpreted as a change of the reflection of S&P 500 signal stability on investors' Bullish sentiment in between the two considered periods, by making the investors' Bullish sentiment less responsive especially in high frequency changes (the blue spots spread across the weeks 900-1837 on Figure 2a). Whereas investors' Bearish sentiment co-movement with S&P 500 in the past period is very similar to the co-movement of investors' Bullish sentiment with S&P 500, the investors' Bearish co-movement with S&P 500 signal in the recent period is significantly larger (the yellow spots spread across the weeks 900-1837 on Figure 2c). This means that investors' sentiment of the Bearish in the recent period has become more responsive to the high frequency changes of the S&P 500 compared to the investors' sentiment of Bullish. In other words, in the recent period (compared to the previous period) there exists an investors' tendency to respond more through their perceived loss than responding through their perceived gain, given the same S&P 500 signal's power spectrum.

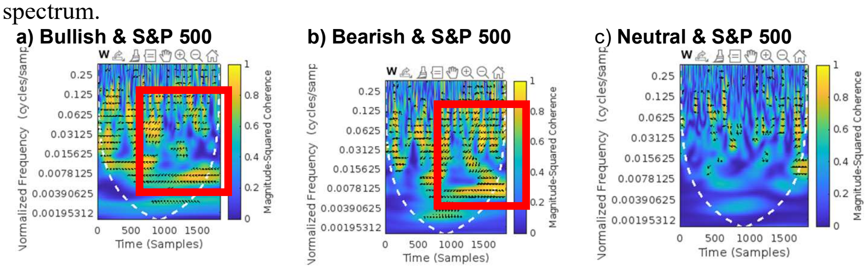

Figure 2. Wavelet coherence (Yellow color represents statistically significant wavelet coherence between signals)

Investors with prospect theory preferences, do not weight perceived gains and losses the same (Yao and Li 2013). According to the results in Figure 2a) and 2b), epistemic utility for perceived gains (Bullish) in the after period of 1000, decreases in relation to the Bullish sentiment's frequency response over time to the frequency changes over time in the S&P 500 signal, which is not the case with the Bearish sentiment's frequency response to the frequency changes of the S&P 500 signal in the after period of 1000. This indicates that the epistemic value of the frequency changes across time of the S&P 500 signal (non-stationarity) represents an additional epistemological input for the epistemological bias more dominant for the Bearish market sentiment. In the context of dynamic prospect theory investment preferences, this means that investors' sentiment above all, it is an epistemologically driven construct, which uses the combined time-frequency domain of the S&P 500 signal to extract epistemic value that would serve to update the sentiments with an emphasized bias on pessimism (losses).

To gain a better insight, we return our analysis to the wavelet transform of the S&P 500's signal (Figure 1). There we can see that in the after 500 weeks period, the S&P 500 signal's frequency domain starts to become widely spread (more uncertain and noisier), which means that the unpredictability of the S&P 500 signal is increasing in the after 500 weeks period on behalf of its frequency distribution (or increasing non-stationarity). On the other hand, according to prospect theory, investors attribute considerably more excessive weight to events with low probabilities compared to the events with high probabilities. If we consider the S&P 500 signal's frequency distribution (across time) as a probability density function of a normal distribution $N(0, \sigma^2)$, the wider the probability density function gets with respect to the σ^2 (such as in our case), the less representative the probability density is. As such, investors attribute considerably more excessive weight to events of wider probability density function compared to events of narrower probability density function.

CONCLUSION

We conclude that time and frequency domains do provide two different epistemological values for investors' sentiment formation whose tradeoff can indeed be explained through a combined time-frequency domain analysis. We analyzed such epistemological differences in terms of the wavelet coherence dependencies between Bullish/Bearish market sentiment and frequency changes of S&P 500's signal across time and results indicated that investors' perceived losses do attribute more epistemic value to the frequency changes (non-stationarity) of S&P 500 signal compared to their perceived gains.

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