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Original scientific paper

BAYESIAN FORECASTS COMBINATION TO IMPROVE THE ROMANIAN INFLATION PREDICTIONS BASED ON ECONOMETRIC MODELS

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Abstract
There are many types of econometric models used in predicting the inflation rate, but in this study we used a Bayesian shrinkage combination approach. This methodology is used in order to improve the predictions accuracy by including information that is not captured by the econometric models. Therefore, experts' forecasts are utilized as prior information, for Romania these predictions being provided by Institute for Economic Forecasting (Dobrescu macromodel), National Commission for Prognosis and European Commission. The empirical results for Romanian inflation show the superiority of a fixed effects model compared to other types of econometric models like VAR, Bayesian VAR, simultaneous equations model, dynamic model, log-linear model. The Bayesian combinations that used experts' predictions as priors, when the shrinkage parameter tends to infinite, improved the accuracy of all forecasts based on individual models, outperforming also zero and equal weights predictions and naïve forecasts.

Keywords: Bayesian forecasts combination, forecasts accuracy, prior, shrinkage parameter, econometric model.

Jel Classification: E37, C51, C52, C53

INTRODUCTION

The inflation targeting needs accurate forecasts of the inflation rate in order to have a successful implementation of the monetary policy. Therefore, it is necessary to know some predictions methodologies that describe specific evolution of the inflation rate. Most of the central banks do not use only some individual models, but also suitable combined forecasts based on these models. In literature many researchers established that the combination of individual models forecasts outperform the predictions based on a single model. In the context of the economic crisis, Julio Roman and Bratu

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Simionescu (2013) showed that the reduction of forecasts uncertainty should be one of the major objective of experts in forecasting. The lower uncertainty of forecasts will improve the decisional process at macroeconomic level, but Terceno and Vigier (2011) showed that the business decisions are also improved.

An important review regarding the forecasts combination was made by Timmermann (2006). Diebold and Pauly (1990) have proposed a Bayesian shrinkage methodology in order to include prior information for improving the predictive accuracy of the combined forecasts. Authors like Wright (2008) or Koop and Potter (2003) used as prior mean zero-weights or equal-weights. Gomez, Gonzalez and Melo (2012) proposed a rolling window estimation method for co-integrated data series of order one in order to calculate the Bayesian weights.

In Romania, the National Bank uses a complex model for short and medium-run predictions. However, the central bank did not make a combination based on Bayesian approach in order to improve the accuracy of its forecasts. Therefore, the object of this article is to make predictions of the inflation rate in Romania using the own econometric models, but also utilizing the Bayesian combination technique in order to improve the accuracy of individual expectations. After a brief description of the methodology, an empirical application is proposed for The Romanian inflation rate forecasts. All the individual models are valid econometric models of the inflation rate, being proposed by us. In this study we will use a prior mean that takes into account the forecasts based on Dobrescu macro-model, which is actually the first international model recognized for the Romanian economy.

METHODS

We consider a number of m h-step-ahead forecasts of the variable denoted by y_t: $f^1_{t/t-h}, \dots, f^m_{t/t-h}$. Granger and Ramanathan (1984) proposed the following forecasts combination:

$$y_t = \alpha' f_{t/t-h} + \varepsilon_t \quad (1)$$

$\alpha = (\alpha_0, \alpha_1, \dots, \alpha_m)'$- vector of regression coefficients

$f_{t/t-h} = (1, f^1_{t/t-h}, \dots, f^m_{t/t-h})'$- vector that contains the intercept and a number of m forecasts (this vector dimension is m+1)

The intercept is introduced to ensure that the bias correction of the combined prediction is optimally determined.

Diebold and Pauly (1990) developed a method for introducing prior information in the regression of forecasts combination by using the g-prior model proposed by Zellner (1986). The model error is independently, normally and identically distributed of average 0 and variance σ^2. Moreover, it is used a natural conjugate normal-gamma prior:

$$p_0(\alpha, \sigma) \propto \sigma^{-m-v_0-2} \exp\{-\frac{1}{2}\sigma^2[v_0 s_0^2 + (\alpha - \alpha^*)' M (\alpha - \alpha *)]\} \quad (2)$$

The form of likelihood function is:

$$L(\alpha,\frac{\sigma}{Y}, F) \propto \sigma^{-T}\exp\{-\frac{1}{2}\sigma^2(Y-F\alpha)'(Y-F\alpha)\} \quad (3)$$

$Y=(y_1,\dots,y_{t-h})'$

$F=(f_{1/1-h},\dots,f_{t-h/t-2h})'$

The marginal posterior of α is:

$$p_1\left(\frac{\alpha}{Y},F\right) \propto [1+\frac{1}{v_1}(\alpha-\bar{\alpha})'s_1^{-2}(M+F'F)(\alpha-\bar{\alpha})]^{-\frac{m+v_1+1}{2}} \quad (4)$$

The marginal posterior mean is:

$$\bar{\alpha}=(M+F'F)^{-1}(M\alpha*+F'F\hat{\alpha}) \quad (5)$$

where:

$v_1=T+v_0$

$s_1^2=\frac{1}{v_1}[v_0s_0^2+Y'Y+\alpha*'M\alpha*-\bar{\alpha}'(M+F'F)\bar{\alpha}]$

$\hat{\alpha}=(F'F)^{-1}F'Y$

Diebold and Pauly (1990) showed the validity of the following relationship for g-prior analysis (M=gF'F):

$$\bar{\alpha}=\frac{g}{1+g}\alpha*+\frac{1}{1+g}\hat{\alpha} \quad (6)$$

g$\in[0,\infty)$ is the shrinkage parameter. This parameter controls the relative weight between the maximum likelihood estimator and the prior mean in the posterior mean.

Wright (2008) utilized zero weight as the prior mean, while Diebold and Pauly (1990) recommended the equal weights. Geweke and Whiteman (2006) specified the prior distribution in Bayesian forecasting by including forecasters (experts) information. In this study we will use as prior weights the estimated parameters of the regression between the forecasters' h-step predictions $f_{t/t-h}^{expert}$ and the forecasts based on different econometric models. The prior mean is:

$$f_{t/t-h}^{expert}=\alpha_t'f_{t/t-h}+\varepsilon_t \rightarrow \alpha*_t=(F'_{t-w+1,t},F_{t-w+1,t})^{-1}F'_{t-w+1,t}F_{t-w+1,t}^{expert} \quad (7)$$

where:

$F_{t-w+1,t}=(f_{t-w+1/t-h-w+1},\dots,f_{t/t-h})'$

$F_{t-w+1,t}^{expert}=(f_{t-w+1/t-h-w+1}^{expert},\dots,f_{t/t-h}^{expert})$

For non-stationary data series, Coulson and Robins (1993) used a linear model to construct the combination technique:

$$y_t-y_{t-h}=\alpha'\widetilde{f_{t/t-h}}+\varepsilon_t \quad (8)$$

$$\tilde{f}_{t/t-h}=(1,f_{t/t-h}^1-y_{t-h},\dots,f_{t/t-h}^m-y_{t-h})' \quad (9)$$

$$f_{t/t-h}^{expert}-f_{t-h/t-2h}^{expert}=\alpha_t'\bar{\bar{f}}_{t/t-h}+\varepsilon_t\ ,$$

where:

$$\bar{\bar{f}}_{t/t-h} = (1, f^1_{t/t-h} - f^{expert}{}_{t-h/t-2h}, \ldots, f^m_{t/t-h} - f^{expert}{}_{t-h/t-2h})' \quad (10)$$

In the following table, Table 1, the extreme cases of the posterior mean are presented, according to the methodology of Coulson and Robins (1993).

Table 1. Posterior mean of the extreme cases

Prior	$g \to \infty$
Experts' predictions	Experts' weights
Equal weights	Equal weights
Zero weights	Random walk weights

For zero weights prior, when g tends to infinite, the posterior mean is actually a zero weight vector. This implies a naïve forecast. The Bayesian approach with equal and zero weights priors supposes that the combination uses the forecasters' expectation as covariate.

EMPIRICAL RESULTS FOR ROMANIAN INFLATION PREDICTIONS

The experts' forecasts are those based on Dobrescu macromodel for the Romanian economy. The available data for inflation rate predictions from 1997 to 2012 are divided into two samples. The first sample (1997–2009) is utilized in estimating the forecast combination model, while the second sample (2010–2012) is useful for assessing the performance of individual models and of the models' combination.

Some accuracy measures are used to compare the forecasts accuracy (mean error-ME, mean absolute error- MAE, root mean square error- RMSE, U1 Theil's statistic and U2 Theil's statistic. First of all, we proposed some individual econometric models used to predict the inflation rate in Romania.

The Phillips curve cannot be observed for data series available for Romania. However, a valid log-linear model was put in evidence:

$$\ln(inflationf_t) = 0.758 + 0.324 \cdot unemployment_t \quad (11)$$

In order to eliminate the inconvenient of a small set of data, the parameters of a log-linear model were estimated by bootstrapping, the residuals being resampled with a number of 10 000 replications:

$$\ln(inflation_t) = 0.756 + 0.324 \cdot unemployment_t \quad (12)$$

In the case of a log-linear model, the coefficient of X variable has the significance of elasticity, while the slope is the product between elasticity and the ratio Y/X. Concretely, at each increase with one unit of the variable X, the dependent variable Y changes in average with 0.3248 or 32.48%. So, at each increase with one per cent in the unemployment rate, the inflation rate increases with 0.3248 percentage points. For this model the other assumptions are tested. The Durbin-Watson tests and Breusch-Godfrey test for a lag equalled to 1 indicated an errors' autocorrelation of order 1. The residuals are homoscedastic, according to White test.

A multiple regression model is estimated, adding as explanatory variable beside the unemployment rate the USD/ROL average exchange rate. The data series for exchange rate is not stationary being necessary a differentiation of order 1. The influence of inflation rate is eliminated from the evolution of this indicator, resulting the real exchange rate. The multiple regression model is estimated using bootstrapped coefficients. The errors are homoscedastic and the auto-correlation is ignored.

$$inflation_t = -105.04 + 20.42 \cdot unemployment_t - 0.0022 \cdot d\_excange\ rate_{t-1} \quad (13)$$

The following simultaneous equations model is considered:

$$inflation_t = a + b\ unemployment_t + c\ exchange\ rate_t + u_t \quad (14)$$

$$exchange\ rate_t = d + e\ exchange\ rate_{t-1} + v_t \quad (15)$$

$exchange\ rate_t$- real exchange rate at time t
$inflation_t$- inflation rate at time t
$unemployment_t$- unemployment rate at time t
$unemployment_t, exchange\ rate_t$- endogenous variables
$unemployment_t, exchange\ rate_{t-1}$- exogenous variables

The type of simultaneous equations model is set up in order to choose the suitable estimation model. The model is over identified, because the first equation is exactly identified while the second one is over identified. The first equation is exactly identified, because the number of absent variables in the equation is 1, a number that equals the number of endogenous variables in the model minus 1 (2-1=1). The second equation is over identified, the number of absent variables in the second equation being greater than the number of endogenous variables minus 1 (2-1=1). The model being over identified, the estimation method is two stages ordinary least squares.

Stage 1: the endogenous variable $exchange\ rate_t$, which is endogenous in the second equation, but exogenous variable in the first equation is regressed according to the exogenous variables in the model ($unemployment_t, exchange\ rate_{t-1}$).

$$exchange\ rate_t = \alpha + \beta\ unemployment_t + \gamma\ exchange\ rate_{t-1} + w_t \quad (16)$$

According to F test, the models is valid, on overall the coefficients of independent variables are statistically significant. The Breusch-Godfrey test shows independent errors.

Stage 2: $exchange\ rate_t$ is introduced with the estimated values in the first equation.

$$inflation_t = a + b\ unemployment_t + c\ \widehat{exchange}\ rate_t + u_t \quad (17)$$

The simultaneous equations model will be denoted by SEM.

For the ARMA model is used the first differentiated inflation rate data series which is stationary, the ADF (Augmented Dickey Fuller) showing the existence of one unit root for the level data set. The best model is an ARMA(1,1) for first differentiated inflation rate.

$$d\_inlationf_t = -0.207 + 0.614 \cdot d\_inflation_{t-1} - 0.99 \cdot \varepsilon_{t-1} + \varepsilon_t \quad (18)$$

According to the following graphs the inverse roots are outside of the unit circle.

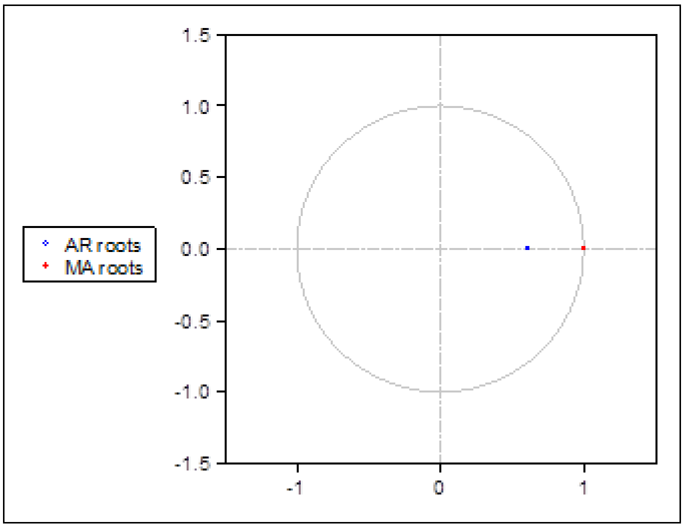

Figure 1. The inverse roots of ARMA polynomial

According to White test, the errors are homoscedastic, not having reasons to reject the hypothesis of homoscedasticity (Prob. greater than 0.05). The study of correlogram shows the errors independence. The Jarque-Bera test indicates that there is not enough evidence to reject the normality distribution of errors (the JB test statistic is 0.98, lower than the critical value of 5.99).

A vector-autoregressive model (VAR model) was proposed for the stationary data series, all of them being differentiated once. All the selection criteria indicate a lag equalled to 4. Portmanteau test indicates the errors autocorrelation, but if the model is used on long run the serial correlation could be ignored. The maximum likelihood test conducts us to the same result of serial correlation. In this case the Prob. being less than 0.05, the null hypothesis is rejected. When Cholesky orthogonalization is used, the residuals follow a normal distribution.

The predictions based on VAR(4) model are used to forecast the original variables on the horizon 2010-2012.

Table 2. Forecasts of inflation (i) (%) and unemployment rate (u) (%) based on VAR(4) models (horizon 2010–2012)

Year	Predictions	
Variable (%)	i	u
2010	4.162	6.98
2011	5.17	5.26
2012	4.71	4.77

For the inflation rate in 2012 the VAR(4) has predicted a pronounced deflation, which was not the case in reality. Even if the inflation decreased in 2012 compared to 2011, it still had got a positive rate.

It is made the assessment of impulse response functions, the inflation, unemployment and exchange rate differentiated data series being denoted by DY, DX and DZ.

It was chosen the variant of multiple graphs for the response of standard errors setting Monte Carlo method with 1000 replications and 10 periods. If the variables order is changed, it is modified the scheme of shocks identification and the impulse-response function and the decomposition of errors variance.

The variance decomposition shows that inflation volatility is mostly due to the evolution of this indicator, but its influence decreases in time, from lag 1 to 10. Till the lag 3, the unemployment rate volatility is explained by the inflation influence, but then, till the end, the contribution of the exchange rate is more significant, more than 50% of the unemployment volatility being explained by the exchange rate. For the exchange rate, more than 65% of its volatility in each period is explained by the same indicator, even if the unemployment rate has a rather high influence (more than 32% in each period).

The unemployment rate is cause of inflation evolution, while overall the exchange rate and the unemployment rate influence the inflation rate. According to Granger causality test, the inflation and the exchange rate influence the unemployment evolution.

We estimated in Matlab BVAR(4) models using Minnesota and non-informative priors. For making the Bayesian estimation, direct and repeated predictions based on BVAR(4) models with intercept and the impulse-response analysis are elaborated, adapting the Matlab program used by Koop and Korobilis (2010) for stationarized data sets of inflation, unemployment and exchange rate (di, ds, dc). The BVAR model depends on the forecasting method. For repeated predictions, the model's form is: Y(t) = A0 + Y(t-1) x A1 + ... + Y(t-p) x Ap + e(t), p number of lags for Y (from 1 to p). The predictions with h steps use the model: Y(t+h) = A0 + Y(t) x A1 + ... + Y(t-p+1) x Ap + e(t+h). In both situations, the model is formulated as: Y(t) = X(t) x A + e(t), where e(t) ~ N(0,SIGMA),a- vector with a=vec(A).

The data are represented as a T*M matrix (T- number of observations, M- number of dependent macroeconomic variables). The X matrix contains all the variables (intercept, exogenous variables and dependent variables with lag.

Table 3. Forecasts of inflation rate (%) based on BVAR(4) models with intercepts (horizon 2010–2012)

Prior		Direct forecasts	Repetitive forecasts
Non-informative prior	2010	1.4545	2.6038
	2011	2.2404	1.1711
	2012	3.8193	2.5614
Minnesota prior	2010	1.4763	1.7266
	2011	0.9807	2.1508
	2012	2.5972	1.2792

For direct forecasts based on non-informative priors an evident increase of the inflation rate is observed for the period 2010-2012. For the rest of the predictions a tendency of increase and decrease cannot be identified, the evolution of the predictions being irregular.

A panel data approach is applied in order to make forecasts of the inflation rate. The data are represented by the effective values of the inflation rate and unemployment rate in Romania and the predictions of 3 institutions: Institute for Economic Forecasting, National Commission for Prognosis and the European Commission during 2001-2012.

The form of the regression model:

$$inflation_t = c + b \cdot prediction\_inflation_{it} + d \cdot prediction\_unemployment_{it} + e \cdot unemployment_t + a_i + \varepsilon_{it} \quad (19)$$

$inflation_t -$ effective inflation rate in year t
$unemployment_t -$ effective unemployment rate in year t
$prediction\_i_{it} -$ the predicted inflation rate made by the institution i for year t
$prediction\_u_{it} -$ the predicted unemployment rate made by the institution i for year t
$a_i -$ individual effects
ε_{it}- random error

First of all we have to decide if we should use an usual regression model or a panel data approach. The OLS estimator is biased and inconsistent, the individual effects being present. The specialists' ability, the type of model could be causes of the differences between predictions. The application of Hausman test was made in order to decide if the model with fixed effects is better than the one with random effects or else. The probability associated to Hausman statistic is less than 0.05, the fixed effects model being better. In this case two important assumptions were checked and the errors are homoscedastic and non-auto-correlated. However, we have to take into account the economic reasons for this type of model. The three fixed effects models are denoted by FEM1, FME2 and FEM3.

Starting from these individual predictions, the combined forecasts were built. The shrinkage parameters are 0, 1 and $g \to \infty$. We used a prior based on experts' expectations, but also zero-weight and equal-weight priors.

The forms of log-linear model and dynamic model are the following:

$$\ln(inf_t) = 0.758 + 0.324 \cdot ur_t \quad (20)$$

$$inf_t = -105.04 + 20.42 \cdot ur_t - 0.0022 \cdot d\_er_{t-1} \quad (21)$$

For BVAR models we will use the following notations:

- BVAR1(4) model for non-informative prior and direct forecasts;
- BVAR2(4) model for non-informative prior and repetitive forecasts;
- BVAR3(4) model for Minnesota prior and direct forecasts;
- BVAR4(4) model for Minnesota prior and repetitive forecasts.

The combined models are built in the following variants:

- CM1 (combined model for g=0, where prior is experts' predictions with equal weights);
- CM2 (combined model for g=0, where prior is represented by equal weights variant);
- CM3 (combined model for g=0, where prior is with null weights);
- CM4 (combined model for g=1, where prior is experts' predictions);
- CM5 (combined model for g=1, where prior is with equal weights);
- CM6 (combined model for g=1, where prior is with null weights);
- CM7 (combined model for $g \to \infty$, where prior is experts' predictions);
- CM8 (combined model for $g \to \infty$, where prior is with equal weights);
- CM9 (combined model for $g \to \infty$, where prior is with null weights).

Table 4. The accuracy of inflation one-year-ahead forecasts in Romania

Model type	ME	MAE	RMSE	U1 Theil's statistic	U2 Theil's statistic
Log-linear	2.3572	2.3572	2.6465	0.3333	0.5567
Dynamic	0.2449	1.6487	1.9392	0.1904	0.7759
SEM	-0.4219	1.2640	1.3947	0.1994	1.0594
ARMA	0.8599	1.5899	1.9202	0.2016	0.7390
VAR(4)	0.3893	1.3093	1.4149	0.1427	0.9128
BVAR1(4)	2.5653	2.8915	3.3826	0.4277	0.4296
BVAR2(4)	2.9579	2.9579	3.3704	0.4534	0.4410
BVAR3(4)	3.3853	3.3853	3.8709	0.5503	0.3802
BVAR4(4)	3.3511	3.3511	3.4875	0.5001	0.4182
FEM1	0.2224	1.2370	1.4212	0.1403	1.0586
FEM2	-2.2007	2.4560	2.8649	0.2252	0.5000
FEM3	-0.7787	1.2000	1.4724	0.1327	0.9932
CM1	1.2453	1.2453	1.5062	0.1658	0.9898
CM2	-0.3572	0.8166	1.0788	0.1012	1.3613
CM3	1.8674	1.8674	2.0734	0.1427	0.7246
CM4	-1.2512	1.2512	1.4501	0.1249	0.9951
CM5	-3.8997	3.8997	4.0143	0.2824	0.3625
CM6	-0.2230	1.0810	1.1143	0.1040	1.3187
CM7	-0.1043	0.6644	0.7726	0.0739	0.9269
CM8	-2.2723	2.2723	2.4720	0.1965	0.5882
CM9	0.7373	0.7373	1.2157	0.1251	1.2637

The upper part of the table refers to individual models, the combined forecasts accuracy measures being presented in the lower part. The predictions performance depends on the window size and the range of the shrinkage parameter g. According to mean errors, the combined forecasts with $g \to \infty$ and experts' predictions as prior have the lowest errors in average. An overestimation tendency is observed for this type of combined prognoses. The absolute mean error, root mean square error and U1 Theil's coefficient indicate that this type of combined predictions has the highest performance. Moreover, the value of U2 statistic shows the superiority of these forecasts compared with the naïve expectations. The experts' forecasts are actually more informative. When g=0, the variant with equal weights provided more accurate forecasts while for g=1, zero weights is the best choice. However, for $g \to \infty$ the experts' combined predictions outperformed all the individual and combined models.

For a null value of g, the U1 statistic shows similar performance of the combined forecasts because the prior mean has zero weight in the posterior average. The experts' expectations are computed with less information, which determines a slightly greater value for U1 statistic.

CONCLUSION

In this study, for the inflation predictions based on econometric models we proposed different Bayesian forecasts combinations. The objective was to check if the combined forecasts improve the degree of accuracy of the initial predictions.

As a novelty, the Bayesian combinations were built using as prior the experts' expectations. In this research for the Romanian inflation we used the forecasts based on Dobrescu macromodel and the expectations provided by National Commission for Prognosis and European Commission. The window size was 9 years, shrinkage parameter g equals 0, 1 and $g \to \infty$. The one-step-ahead forecasts were made for a horizon of 3 years. Indeed, the Bayesian combinations that used experts' predictions as priors, when the shrinkage parameter tends to infinite, improved the accuracy of all forecasts based on individual models. However, the Bayesian combined forecasts depend on the window size and the selected predictions.

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